Math 656 • March 26, 2014

Midterm Examination

1) (16pts) Find all values of z in polar or Cartesian form, and plot them as points in the complex plane:

(a) $z = (1+i)^{i}$ (b) $z = \tanh^{-1}(-2)$

2) (18pts) For each integral below, describe *all* singularities of the integrand, and use the most convenient method to calculate the integral. If the integral is zero, explain why.

(a) $\oint\limits_{|z|=1} \frac{dz}{\cosh z}$ (b) $\oint\limits_{|z|=2} \frac{\cos(\pi z)\,dz}{z^2 + z}$ (c) $\oint\limits_{|z|=1} \frac{e^z\,dz}{z^3 + 5i\,z^2}$

3) (14pts) Use the most convenient method to calculate the following integrals over a quarter-circle *C* centered at the origin and connecting point *z*=*i* to *z*=1. Is the integral in part (a) single-valued?

(a) $\int\limits_C z\left(z^2 - 3\right)^{1/2} dz$ (b) $\int\limits_C \left(\overline{z} + z\right) dz$

4) (13pts) Find the bound on $\left| \int\limits_C \frac{\sin z\,dz}{z+i} \right|$, where the integration contour *C* is a semi-circle of radius 2 connecting points *–2* and *2* in the upper half-plane.

Pick any three out of the following problems:

5) (13pts) Find and sketch the image of the half-ring 1<|*z*|<2, 0<arg *z*<π, under the transformation *w* = (*i* +1) log *z* (hint: consider this as a composition/sequence of two elementary transformations)

6) (13pts) Consider the function $f(z) = \mathrm{Re}(z)$. Is this function continuous in any part of complex plane? Is it complex-differentiable anywhere? Examine analyticity directly (using limit definition of derivative), and verify your answer using Cauchy-Riemann equations.

7) (13pts) Consider *any* branch of function $\left(z^2 + 1\right)^{1/3}$, describe its branch cut(s) and describe the discontinuity of this function across the branch cut(s).

8) (13pts) Suppose that $f(z)$ is entire. Use Cauchy-Riemann identities to prove that function $\mathrm{F}(z) = \overline{f(\overline{z})}$ is also entire (hint: relate $F(z) = U(x,y) + iV(x,y)$ to $f(z) = u(x,y) + i\,v(x,y)$)